Technical Report No. 32-678

A Study of Electrical Discharge in Low-Pressure Air

M. Edmund Ellion

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JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

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M. Edmund Elion

William S. Shipley, Chief
Program Engineering Section

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ABSTRACT

A review was made of the available literature describing the mechanisms of electrical breakdown in gases. The mechanisms suggested by various investigators were then modified or expanded to a form suitable for determining the maximum allowable voltage that can be applied, at any given air pressure, across any gap between conductors in a spacecraft. The final results, giving breakdown voltage as a function of gas pressure for typical gap spacings, are presented in graphical and tabular form. These results indicate that, at air pressures of less than $10^{-2}$ mm Hg, electrical discharge will not occur at any voltage. A bibliography of the literature reviewed prior to this study is included.

I. INTRODUCTION

A. Purpose of Study

Air in its normal state is a very poor conductor of electricity. Under certain conditions, however, the air can be ionized and will then be able to pass a high current. This flow of high current between conductors could cause serious damage to the materials in a spacecraft electrical system.

In general, the purpose of the study reported here was to determine the pressure conditions that could cause electrical breakdown in the spacecraft system. Specifically, it was desired to determine the relationship between the electrical-breakdown potential and the pressure of the air. With this information, it should be possible to specify the pressure conditions under which high-voltage electrical power cannot be safely applied to the system. Data of this nature would be of value both for test-chamber simulation experiments and for the programming of actual flights.

B. Types of Gas Discharge

The types of discharge considered in this study are the Townsend discharge, the glow discharge, the corona discharge, and the arc discharge, in both direct-current and alternating-current fields.

The subject of gas discharge has been discussed in numerous publications, as indicated by the extensive Bibliography presented in this Report. However, despite treatment of the subject by these numerous authors, the exact nature of the discharge mechanism is not known. Because of this lack of understanding, there is no theory that can satisfactorily predict the conditions of discharge, and it becomes necessary to rely on experimental data. One of the difficulties in specifying the conditions under which breakdown will occur is that the discharge voltage depends strongly on the exact composition of the gas and, under some conditions, on the composition and condition of the electrodes. In a space vehicle leaving the Earth, there is a decreasing-pressure environment. Within the spacecraft, the pressure and composition of the gas surrounding the electrodes can be substantially
different from the outside-air conditions. The difference in pressure will occur if the compartment is not suitably vented; the difference in gas composition can result from degassing of the insulation, paints, etc., within the compartment. Additionally, the temperature of the gas will differ from the ambient temperature because of gas expansion and heat transfer from the spacecraft. All these factors make it difficult to specify the conditions within the spacecraft power-system compartments. Another difficulty of a more fundamental nature is the serious lack of experimental data, particularly at low pressures, for air contaminated by other gases, and for electrodes other than iron or platinum.

C. Conditions of Interest

A curve of breakdown voltage plotted against air pressure would allow the testor to determine the conditions under which it is safe to turn on power in various systems of the spacecraft. Several factors have made it extremely difficult to obtain such a generalized curve of breakdown voltage. In addition to the practical considerations (that little or no information exists for low-pressure air, and that the breakdown voltage varies appreciably with the gas and electrode composition), there is the inherent problem that breakdown depends on the mass of the gas per unit area between the electrodes (the product of density and electrode spacing) and not merely on the gas pressure.

Despite these difficulties, a curve of breakdown voltage vs air pressure from 1 atm to $10^{-4}$ mm Hg was developed (Fig. 1). When properly used, this graph will serve the desired purposes. The fundamental physical mechanisms of electrical breakdown in air, discussed in Section II, provide the basis for development of this generalized breakdown-voltage curve.

![Fig. 1. Typical circuit for electrical-breakdown studies](image)

II. TECHNICAL DISCUSSION

A. Mechanism of Gas Discharge

In this Report, the terms electrical breakdown or discharge are used to indicate the flow of current through an air gap between two conductors. These two conductors are referred to as electrodes. There are three fundamental types of discharge: Townsend discharge, glow discharge, and arc discharge. It will be seen that the corona discharge and the abnormal glow discharge are forms of the basic glow discharge. A circuit typical of those considered is illustrated in Fig. 2. The discharge gap may be the space between connector pins, between a pin and the case, between conductors on a board, or between any other uninsulated conductors that can act as cathode and anode.
Air is normally a poor conductor of electricity. However, if the air contains a number of electrons and positive ions, it becomes a good conductor. There are three basic ways of generating these electrons from solid surfaces: by heat, by light, or by bombardment with energetic particles. The first two methods, respectively described as thermionic and photoelectric, are not considered in this study because the design of the spacecraft would prevent such emission from taking place. The third mechanism, bombardment, is considered in detail. The overall mechanism is first discussed in general terms, to provide a basis for the subsequent amplified description of the electrical breakdown.

If a voltage is applied to the circuit illustrated in Fig. 2, the current will follow a curve such as that shown in Fig. 3.

![Graph](image)

Fig. 3. Current flow between plane electrodes as a function of gap distance

At low voltages ($V_1$ to $V_2$, Fig. 3), the current rises in an approximately linear manner, similar to that of a constant resistance in Ohm's law. In this region, the current is carried by the few free electrons that are always present in the air. From $V_2$ to $V_4$, the current is substantially constant, since all the available electrons are being utilized and no new ones are being generated. At the voltage $V_4$, the available electrons are accelerated to a velocity that is sufficient to ionize the air on impact and to form additional electrons and positive ions.

For operation in a vacuum, the maximum direct current is limited to the equivalent of the horizontal portion of the curve, except at very high voltages. At these extreme voltages, the electrons can have sufficient energy to generate X-rays or other radiation at the anode which, upon reaching the cathode, could provide the energy to release additional electrons.

In a gas, the current can increase substantially, this change depending principally on the characteristics of the external circuit. It is this increase in current that causes damage, and it is the corresponding value of the breakdown voltage $V_4$ which must be determined for various conditions. At the voltage $V_4$, the positive ions formed in the air have been accelerated toward the cathode with sufficient energy to generate secondary electrons from impact on the metal. These secondary electrons are accelerated toward the anode and generate additional electrons and positive ions upon impact with the air molecules. The result is the rapid increase of current with voltage.

Values of the breakdown voltage depend upon many factors, the most important being the electrode geometry and spacing, the electrode material and the condition of its surface, the gas composition, the condition of the gas, and the ability of any external exciting energy to provide electrons at the cathode. For the circuit shown in Fig. 2, the equation for voltage may be written as

$$ V = V_e + V_d $$  \hspace{1cm} (1)

where

$$ V_e = \text{external voltage drop} $$

$$ V_d = \text{discharge-gap voltage drop} $$

$$ V = IR + L \frac{dI}{dt} + V_d $$ \hspace{1cm} (2)

We are now in a position to consider the discharge gap in detail. Fig. 4 illustrates the current flow between two plane electrodes, expressed by

$$ I_c = I_e + I_p = I_a $$ \hspace{1cm} (3)

![Graph](image)

Fig. 4. Initial impact of electron and air molecule
where

\[ I_c = \text{total electric current at cathode} \]
\[ I_e = \text{current flow in gas due to electrons} \]
\[ I_p = \text{current flow at cathode due to positive gas ions} \]
\[ I_a = \text{total current at anode} \]

Fig. 5 illustrates an initial electron leaving the cathode and impacting with an air molecule. The resulting positive ion and electron proceed to the cathode and anode, respectively. The positive ion and any photons formed may generate additional electrons upon impacting the cathode.

At any distance \( x \) from the cathode, the current in the air gap due to the electrons leaving the cathode may be written (Ref. 1) as

\[ I = I_e e^{\alpha x} \]  

where

\[ I_e = \text{electron current leaving cathode} \]
\[ \alpha = \text{number of ionizing collisions made by an electron in traveling 1 cm (known as the first Townsend coefficient)} \]

For steady state, the current at the cathode \( I_c \) must equal the current at the anode \( I_a \), which is at a distance of \( d \) cm. Thus,

\[ I_c = I_a = I_e e^{\alpha d} \]  

or

\[ I_e = I_c + I_p \]

or

\[ I_p = I_e e^{\alpha d} - I_e = I_e (e^{\alpha d} - 1) \]

In addition to the electrons that have sufficient energy to ionize the air molecules and produce an electron and a positive ion, there are three other sources for the generation of additional negative ions (electrons) or positive ions: (1) the positive ions generated in the gas can be accelerated to the cathode and can, in turn, ionize additional gas molecules or strike the cathode and release secondary electrons from the metal surface; (2) additional electrons can be released from the cathode by the photons given off when the air ionizes; (3) the cathode can release additional electrons when receiving energy from other excited air atoms. Thus, the total number of electrons leaving the cathode will be made up of the primary electrons \( I_c \) and the secondary electrons \( I_e \), that are generated at the cathode by the ions, photons, and excited atoms (Ref. 2).

The current at the anode can be written (Ref. 3) as

\[ I_a = I_e e^{\alpha d} \]  

where

\[ I_{vc} = I_e + I_p = \text{total cathode current} \]

\[ I_{vc} = \gamma I_p \]

\[ \gamma = \text{number of electrons produced per ion impact, photon, etc.} \]

From Eq. (7), with a total electron current at the cathode of \( I_{vc} \),

\[ I_{vc} = \gamma I_p = \gamma I_e (e^{\alpha d} - 1) \]

Substituting Eq. (9) into Eq. (8a),

\[ I_{vc} - I_e + \gamma I_{vc} (e^{\alpha d} - 1) \]

Solving for the relation of \( I_{vc} \),

\[ I_{vc} = I_e [1 - \gamma (e^{\alpha d} - 1)]^{-1} \]

Substituting this value into Eq. (8),

\[ I_e = \frac{I_e e^{\alpha d}}{1 - \gamma (e^{\alpha d} - 1)} \]

From Eq. (11), it can be seen that the current at the anode becomes infinite if the denominator becomes equal to zero, or if

\[ \gamma (e^{\alpha d} - 1) = 1 \]
In reality, the current will never become infinite, since it is limited by the external-circuit characteristics. The values of \( \alpha \) and \( \gamma \) are functions of both the applied voltage and the pressure of the gas. The coefficient \( \alpha \) also depends on the composition of the gas.

The relation derived for the breakdown condition in Eq. (12) is similar to the relations developed empirically by Townsend and also by Paschen (Ref. 2).

Substituting Eq. (9) into Eq. (8a) and (8) has produced a relation identical with the Townsend equation under some conditions. The Townsend equation also gives the total current flow as a function of the initial current \( I_e \) that may result from a few free electrons in the gas, as in our previous derivation. The two empirical coefficients, \( \alpha \) and \( \beta \), are called the first and second Townsend coefficients. The Townsend relation is

\[
I = I_e \frac{\alpha - \beta}{\alpha - \beta e^{\alpha - \beta x}}
\]

For low pressure, \( \beta \ll \alpha \) and, at the anode, \( x = d \). The anode current at lower pressure then becomes

\[
I_a = I_e \frac{\alpha e^{\alpha d}}{\alpha - \beta} = I_e \frac{e^{\alpha d}}{1 - \frac{\beta}{\alpha} e^{\alpha d}}
\]

If \( \beta/\alpha \) is replaced by \( \gamma \), then Eq. (13) is the same as Eq. (11) for \( e^{\alpha d} \gg 1 \).

It should be noted that the derived Eq. (12) applies to a steady state, since we assumed that the cathode current equals the anode current. The electrical breakdown, however, is a transient phenomenon. Fortunately, the relation for breakdown given in Eq. (12) can be obtained directly from the definition of \( \gamma \) and \( \alpha \), since the number of ions produced in the gas by a single electron is given by \( e^{\gamma d} \), and the number of electrons generated at the cathode for each ion generated in the gas is given by \( \gamma \). Thus, the product \( \gamma e^{\alpha d} \) is the number of electrons generated at the cathode. If \( \gamma e^{\alpha d} = 1 \), we have one electron generated at the cathode which will generate \( e^{\gamma d} \) ions in the gas, which, in turn, will generate one electron at the cathode. Thus, the process is barely self-sustaining. If the condition is exceeded by any amount, then the number of electrons generated at the cathode will multiply during each cycle, and an avalanche of electrons will be formed to cause breakdown.

The relation obtained empirically by Paschen can be derived from our breakdown criteria with some other experimental information. The probability coefficient \( \gamma \) has been experimentally determined to vary as a function of the ratio \( E/p \), where \( E \) is the field strength and \( p \) is the pressure of the gas in the gap. Similarly, the coefficient \( \alpha \) is known to vary as a function of the product of pressure and the ratio \( E/p \). These conditions can be written as

\[
\gamma = f_1 \frac{E}{p}
\]

\[
\alpha = p f_2 \frac{E}{p}
\]

With these relations, Eq. (12) can be written for \( \gamma e^{\alpha d} = 1 \) as

\[
f_1 \frac{E}{p} e^{\gamma d} f_1 \left( \frac{E}{p} \right) = 1
\]

or

\[
f_1 \frac{Ed}{p} e^{\gamma d} f_1 \left( \frac{Ed}{p} \right) = 1
\]

and, for a uniform field, the product of field strength and spacing is the voltage across the gap \( Ed = V \). Then

\[
f_1 \frac{V}{pd} e^{\gamma d} f_1 \left( \frac{V}{pd} \right) = 1
\]

The solution to this equation is the Paschen law relation, developed empirically in 1889:

\[
V = f_3 (pd)
\]

Since the basic mechanism of breakdown has now been developed, it is possible to discuss the various types of discharge and the conditions under which these discharges may occur.

### B. Arc and Glow Discharge

Consider again the circuit shown in Fig. 2. As the voltage is increased, the current will follow a curve such as that shown in Fig. 3. This curve has been redrawn in Fig. 6 to show also the current-voltage variation in the glow-discharge and arc-discharge regions. When the voltage across the gap equals \( V_1 \), it is assumed that the gas pressure and gap spacing are suitable to allow breakdown. Whether an arc or a glow discharge forms depends on the amount of current that is allowed to flow through the external circuit. It should be noted that the breakdown voltage is substantially greater than the voltage required to sustain either the arc or the glow discharge.
A gas will become locally conductive when the field strength exceeds a certain value which is dependent on the condition and composition of the gas and independent of the electrode geometries. The field strength, however, which depends on the voltage and the electrode geometries, is the gradient of the voltage in the direction of the anode. It will be useful to develop the field-strength relation for concentric cylinders.

The field strength, by definition, may be written as

\[ E = \frac{dV}{dr} \]

Using Ohm's law and the continuity equation for current gives

\[ V = IR = I \int_{r_o}^{r} \frac{dr}{2\pi r} = -\frac{\rho}{2\pi} \ln \frac{r_o}{r} \]

where \( \rho \) is resistivity.

Then

\[ V = -\frac{\rho}{2\pi} \ln \frac{r_o}{r} \]

and

\[ E = \frac{dV}{dr} = \frac{\rho}{2\pi} \frac{V}{r \ln \frac{r_o}{r}} \]

Now we may consider a nonuniform field that exists between two concentric cylinders, such as a small wire inside a comparatively large cylinder. We have shown that the field strength at any radius \( r \) can be written as \( E = V/\left[r \ln (r_o/r)\right] \), where \( V \) is the applied voltage across the wire and cylinder, and \( r_o \) is the outer-cylinder radius. When the voltage is increased to a certain value, the gas will become conductive at the region of highest field strength. This conducting layer acts like an increased wire diameter, and the field strength at the boundary of the conducting layer will change, as indicated by the field-strength equation. It will be of interest to plot the field strength as a function of the radius. The minimum point in the curve may be found by setting the field-strength gradient equal to zero, as shown below:

\[ \frac{dE}{dr} = \frac{d}{dr} \left( \frac{V}{r \ln \frac{r_o}{r}} \right) = \frac{V}{r^2 \left( \ln \frac{r_o}{r} \right)^2} - \frac{V}{r^2 \left( \ln \frac{r_o}{r} \right)} = 0 \]

or

\[ \frac{r}{r_o} = (e)^{-1} \]
Fig. 7. General curves of breakdown voltage: (a) as a function of gas pressure times gap spacing; and (b) as a function of gap spacings of 2.0 and 0.1 cm

Figure 7 shows the variation of field strength with radius for concentric cylinders. The corona will grow until the conducting-layer radius increases to the point where the field strength has decreased below the value required to cause conduction in the air. From this conductive and essentially equipotential region, there will be extremely short-lived streamers that transfer electrons from the corona to the electrode. If the value of the minimum applied field in Fig. 6 is still greater than the value required to make the gas conductive, then the corona will grow to fill the entire gap and result in either a glow discharge or an arc. It is interesting to note that a corona can never occur between parallel plates, where \( E = V/d \); if it should start to form and decrease the value of the spacing \( d \), the field strength would increase, rather than decrease, and thus the corona would grow rapidly to the other electrode, causing complete breakdown into a glow or arc discharge. One other similar observation regarding concentric cylinders should be noted. The ratio of wire radius to outer-cylinder radius must be less than \( 1/e \) or \( 1/2.718 \), in order to produce a corona. This condition is evident in Fig. 6. Otherwise, an increase in the inner radius, caused by the formation of a conducting layer, would lead to an increase in the field strength and result in complete breakdown into an arc or glow discharge.

We have seen that a corona can form only where the field strength is decreasing within the gap; otherwise, the discharge would continue to grow to the anode and would no longer be a corona. Other geometries that can sustain a corona include two points, two cylinders (not necessarily concentric), a cylinder and a plane, or any other electrode configuration that produces a field strength decreasing with distance from the cathode. The field strength for any of these geometries can be easily derived by following the method given for concentric cylinders. The field strength that will cause breakdown is the same, regardless of the electrode geometries. If the value is known for concentric cylinders, the breakdown potential for any geometry can be obtained by using the same value of field strength in the appropriate field-strength equation and solving for the potential.

### D. High-Frequency Discharge

When the direct-current field is replaced with an alternating-current field, the basic mechanism of gas discharge does not change if the dimensions of the gap are small relative to the wavelength of the exciting voltage, and if the gas pressure is sufficiently high that the mean free path of an electron is much less than the gap spacing.

The mean free path of an electron in air at a pressure \( p \) and a temperature \( T \) is, from Ref. 1,

\[
\lambda = \frac{0.02}{p} \frac{273}{T}
\]

Equation (15) gives the mean free path of an electron equal to \( 2.63 \times 10^{-5} \) cm for a pressure of 760 mm Hg and a temperature of 273°K. The mean free path increases to 2 cm at a gas pressure of \( 10^{-2} \) mm Hg. These relations are used further in Section II-E, where we develop the generalized curve.

The wavelength of the exciting voltage can be written as

\[
l = \frac{c}{\omega}
\]

where

- \( c \) = velocity of light
- \( \omega \) = frequency of excitation

If the geometric spacings vary from 2 to 0.1 cm, then the maximum value of frequency that can be treated as a steady applied voltage becomes \( 10^{10} \) cps, according to Eq. (16), for a wavelength of 3 cm.

The other consideration in determining frequency effects on the breakdown voltage is the relation between the time at which breakdown occurs and the rise and fall of the exciting voltage. Since ions move fairly slowly, a time of the order of \( 10^{-5} \) sec is required for the ion to reach the cathode; the photons travel with the velocity of light and require a time of the order of \( 10^{-10} \) sec; and the metastable atoms move by diffusion alone, thus...
requiring times of the order of $10^{-3}$ sec. These considerations indicate that, if the time for 1 cycle of the imposed voltage is greater than $10^{-3}$ sec, representing a frequency of under $10^3$ cps, then the breakdown voltage should be the same as that for a steady-state direct voltage. It appears that the ion and photon mechanisms are more important in producing secondary electrons; thus, the frequencies that can be treated as direct-current applied fields will vary between $10^3$ and $10^{10}$ cps. If the frequency exceeds these values, then the breakdown voltage will exceed the direct-current values. We are interested in determining the maximum allowable voltage before breakdown and, consequently, are interested in the minimum breakdown voltage; hence, to be conservative, we should use the direct-current values when in doubt.

If the pressure in the gas drops so low that the mean free path of the electrons becomes of the order of the electrode spacings, then the avalanche type of breakdown cannot occur. In this case, breakdown would occur because the electrons impact the anode with sufficient energy to release radiation such as X-rays which, in turn, would impact the cathode to release additional secondary electrons. This mechanism would require an extremely high voltage equal to the vacuum breakdown voltage and, thus, is of no concern in this study.

**E. Generalized Breakdown Curve**

In the preceding discussion, we have introduced several factors that make it difficult to define a generalized breakdown curve. It was noted that the breakdown voltage depends on the following principal variables: gas composition, temperature, and pressure; electrode composition, surface condition, spacing, and geometry. In addition, the type of breakdown depends on the applied voltage and the external electric circuit. In order to obtain a useful engineering type of curve, it is obvious that we must make some simplifying assumptions. These assumptions must be carefully considered to ensure that the generalized curve will indicate safe operating conditions. On the basis of the mechanisms discussed above, we now present a simplified model and develop the generalized curve.

The following conditions were assumed as representing values of extreme interest:

1. Spacecraft-compartment air pressure: 760 to $10^{-7}$ mm Hg
2. Minimum wire size: 22 gauge
3. Maximum spacing between electrodes (e.g., circuit-board conductors, etc.): 2.0 cm
4. Minimum spacing between electrodes (e.g., connector pins, etc.): 0.1 cm

From these data, it is possible to estimate the minimum air pressure for which the electrical-avalanche breakdown mechanism will occur. Remembering that the breakdown mechanism requires the emitted electron to hit an air molecule, generating more electrons which, in turn, ionize other air molecules, we can specify that the mean free path of the electron must be less than the electrode spacing (i.e., $\lambda \ll d$). For the extreme case, we set the mean free path of the electron equal to the gap space, in order to obtain a conservative value for the breakdown voltage. Thus, from Eq. (15),

$$\text{Minimum breakdown pressure} = 0.2 \text{ mm Hg}$$
$$\text{for } d = 0.1 \text{ cm}$$

$$\text{Minimum breakdown pressure} = 0.01 \text{ mm Hg}$$
$$\text{for } d = 2.0 \text{ cm}$$

The general curve of breakdown voltage as a function of the product of gas pressure and gap spacing is shown in Fig. 8(a). A family of curves is then obtained for breakdown voltage vs gas pressure for various gap spacings, as illustrated in Fig. 8(b).

![Fig. 8. Composite curve of breakdown voltage vs gas pressure for gap spacings of 2.0 and 0.1 cm](image)

For operation to the left of the minimum point on the curves, a smaller gap or a lower pressure makes a better insulator. For operation to the right of the minimum point, a larger gap or a higher pressure makes a better insulator. We are interested only in the maximum allowable voltage for the specified conditions. Thus, Fig. 8(b) can be simplified further by using the right-hand side of the curve for $d = 0.1$ cm down to the minimum voltage, together with the left-hand side of the curve for $d = 2.0$ cm, as illustrated in Fig. 9.
It should be noted that there is a minimum value of voltage below which no discharge will occur for any value of pressure, regardless of how small the gap may become. At greater pressures, the mean free path of the electron is lower; consequently, a voltage greater than the minimum value is necessary to accelerate the electron to the kinetic energy required to cause ionization. At lower pressures, there are fewer air molecules, and thus the probability of collision is decreased. In order to obtain the necessary ionizations, the electrons must have a higher kinetic energy, implying that the breakdown voltage is above the minimum value.

The minimum pressure at which an avalanche breakdown can occur is determined by setting the mean free path of the electron equal to the 2.0-cm gap spacing. From Eq. (15), with a temperature of 273°K, we obtain

\[ p_{\text{min}} = \frac{0.02}{2.0} = 10^{-2} \text{ mm Hg} \]

Having arrived at a technique for presenting the breakdown voltage as a function of the air pressure alone, it is now necessary only to specify the values of voltage at various pressures. The various investigators of this problem have selected different values for voltage breakdown. This variation could be due to small changes in the types of electrodes, the surface conditions of the electrodes, or the composition of the air. The values that appear most reliable were taken from Refs. 1, 4, 5, and 6. These data are for dry air between iron or platinum electrodes. As discussed above, there are few data at low pressures. Also, it is to be noted that each set of data is valid only for the electrodes used in a specific test. An electrode having a small work function may withstand only a fraction of these voltages before producing breakdown, depending upon the ease with which the metal releases electrons (the coefficient \(\gamma\) in Eq. 9). There seems to be no simple relation between the coefficient \(\gamma\) and the work function, except the obvious one that the kinetic and potential energy of the ion must exceed the work function to release an electron (actually, they must have a value \(2e\) times the work function). For convenience, Table 1 presents a list of electron work functions for the elements, taken from Ref. 7.

For corona discharge, a useful relation is given in Ref. 6:

\[ V = 21.7 \, D_c \left(1 + \frac{136}{pD_w} \right) \ln \left( \frac{D_w}{D_c} \right) \]

where

\( D_c \) = diameter of cylinder, cm
\( V \) = breakdown voltage, v
\( D_w \) = diameter of wire, cm
\( p \) = air pressure, mm Hg

Reference to Fig. 7, with the accompanying discussion, shows that this equation is valid as long as \(D_w/D_c > 2.718\). Outside this range, the mechanism changes from corona discharge to an arc or glow discharge. The minimum corona-discharge voltage will occur, according to Eq. (16), when \(D_w\) is large and \(D_c\) is small. If the minimum allowable value of \(D_w/D_c\) is 2.718, and the largest gap spacing

<table>
<thead>
<tr>
<th>Element</th>
<th>Preferred values of work function, ev</th>
<th>Element</th>
<th>Preferred values of work function, ev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ag</td>
<td>4.5</td>
<td>Ge</td>
<td>4.3</td>
</tr>
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<td>Hf</td>
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<td>C</td>
<td>4.3</td>
<td>Mn</td>
<td>3.8</td>
</tr>
<tr>
<td>Ca</td>
<td>2.7</td>
<td>Mo</td>
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<td>Nd</td>
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<tr>
<td>Cr</td>
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<td>Ni</td>
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<tr>
<td>Co</td>
<td>4.4</td>
<td>Os</td>
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</tr>
<tr>
<td>Cs</td>
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<td>Pb</td>
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</tr>
<tr>
<td>Cu</td>
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<td>Rb</td>
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<td>Ga</td>
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<td>Re</td>
<td>5.1</td>
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</table>

*The values in this Table (taken from Ref. 7) should be considered approximate, not only because of the difficulty of measurement but also because of the anisotropy and allotropy of the work function."
under consideration is 2.0 cm, then the largest wire diameter will be 2.32 cm. These conditions will give the minimum voltage that can form a corona discharge. A tabulation of breakdown voltage appears in Table 2 for $D_o = 6.32$ cm and $D_w = 2.32$ cm.

The empirical equation used to calculate these data is not valid below a few millimeters of mercury, and no experimental data could be found for air at lower pressures. Until further experimental results become available, it is recommended that the breakdown voltage for the arc or glow discharge be used as the maximum allowable voltage.

Table 3 presents the minimum values for the arc- or glow-discharge type of breakdown, obtained by using a gap spacing of 2.0 cm for the left side of the curve and 0.1 cm for the right portion, with a minimum allowable pressure of $10^{-2}$ cm.

**Table 2. Corona-discharge breakdown voltages at various air pressures for $D_o = 6.32$ cm and $D_w = 2.32$ cm**

<table>
<thead>
<tr>
<th>Air pressure $p$, mm Hg</th>
<th>Breakdown voltage for corona discharge, kv</th>
</tr>
</thead>
<tbody>
<tr>
<td>760</td>
<td>47.2</td>
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<tr>
<td>500</td>
<td>34.0</td>
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<td>300</td>
<td>21.8</td>
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<tr>
<td>200</td>
<td>15.5</td>
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<tr>
<td>100</td>
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<td>10</td>
<td>1.7</td>
</tr>
<tr>
<td>1</td>
<td>0.4</td>
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</tbody>
</table>

**Table 3. Arc- or glow-discharge breakdown voltages at various air pressures**

<table>
<thead>
<tr>
<th>Air pressure $p$, mm Hg</th>
<th>Breakdown voltage for arc or glow discharge, kv</th>
</tr>
</thead>
<tbody>
<tr>
<td>760</td>
<td>10.0</td>
</tr>
<tr>
<td>500</td>
<td>4.0</td>
</tr>
<tr>
<td>300</td>
<td>2.5</td>
</tr>
<tr>
<td>200</td>
<td>1.8</td>
</tr>
<tr>
<td>100</td>
<td>1.0</td>
</tr>
<tr>
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<td>0.65</td>
</tr>
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<td>0.40</td>
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<tr>
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<td>0.33</td>
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<tr>
<td>0.1</td>
<td>0.33</td>
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<tr>
<td>0.01</td>
<td>1.5</td>
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<tr>
<td>Less than 0.01</td>
<td>Vacuum discharge value</td>
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</table>
III. CONCLUDING REMARKS

For the spacing conditions chosen in the present study, a comparison of the corona-discharge voltages and the values for arc or glow discharge indicates that corona discharge always requires a higher voltage; thus, to be conservative, the values for breakdown voltage in arc or glow discharge should be used. On the basis of results obtained in this study, the values given in Table 3 and Fig. 1 are the minimum voltages which can cause breakdown in arc or glow discharge.

It should be noted, however, that these allowable voltages are based on very few experimental data. In future investigations, a rather simple experiment could be performed to verify the prediction that discharge in a 2-cm gap will require the same high voltages as discharge in a vacuum if the gap pressure is less than $10^{-2}$ mm Hg. As illustrated in Fig. 10, the equipment required for this test would include a power source to charge the capacitors, a resistor to protect the power supply, a vacuum pump to evacuate the gap, and a voltmeter and vacuum gage to measure discharge voltage and gas pressure.

![Fig. 10. Schematic of typical test setup for electrical-discharge investigations](image)

The equipment shown in Fig. 10 could, of course, be used to verify the entire voltage-breakdown curve. A test could be made by evacuating the gap to the desired pressure, closing switch S-1, and setting the power supply to the desired voltage. With the capacitor fully charged, switch S-1 would open and switch S-2 would close. This procedure could be repeated at higher values of voltage until a discharge occurred in the gap.

REFERENCES

BIBLIOGRAPHY


