20. COMPRESSOR NOISE ANALYSIS

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SUMMARY

The results of a general theory for the prediction of noise radiation by fans and compressors are described. Emphasis is placed on the basic mechanisms underlying the noise radiation and the leading results from the application of the theory. Preliminary agreement with experiment is shown. The theory is capable of including virtually all the major design variables of a compressor and is thought to be of direct use in trade-off studies to minimize noise at an early stage of engine design.

INTRODUCTION

This paper is concerned with the application of theory to the prediction and the understanding of the noise radiation by compressors and fans. The noise produced by these devices has been increasing in importance as a source of community noise, and the projected introduction of fans with very high bypass ratios, possibly operating at supersonic tip speeds, will lead to the domination of the observed sound field of an airplane by the sound radiation from this source. The objective of the theory is to give an understanding of the basic mechanisms underlying the compressor noise with the hope that it will lead to improved methods for the control and the prediction of noise radiation. One of the key problems facing the engine designer at the present time is that no method is available for predicting the detailed acoustic effects of design changes; therefore, it becomes extremely difficult to make the appropriate design trade-offs to minimize the noise radiation at the initial design stages of any given engine.

Much important work has now been accomplished on compressor noise, both theoretically and experimentally (refs. 1 to 13). A review of the available information on compressor noise is given in reference 11. The work reported herein is a condensation of work reported in more detail in reference 14. In the present paper, only the basic physical mechanisms underlying the compressor noise will be discussed together with the leading results from the application of the theory. For more details of the mathematical methods and physical models used in the theory, the reader is advised to consult reference 14.

The model used in the analysis regards the fan as an isolated rotor or stator in free space on which fluctuating forces are acting. The effect of the upstream or
downstream blade rows are included only in terms of the fluctuating forces which result from their presence. Methods for calculating these fluctuating forces have also been derived.

Several other sources of noise are feasible. Fluctuating thickness and fluctuating mass source terms can be significant (ref. 14). The acoustic stresses acting around the blade can also give rise to both discrete and broadband radiation (ref. 13). There is also a possibility of interactions between various sources inside the compressor giving rise to additional noise terms. However, it is thought that most of these other sources will be negligible, and certainly the mass source terms calculated in reference 14 were found to be very small. Furthermore, the basic phase effects discussed in this paper will apply to any type of acoustic source function so that many of the key results and design trends should be applicable regardless of the specific type of acoustic source which is acting.

It is important to realize the complexity of the compressor noise radiation problem. Figure 1 shows the cause and effect chain which leads to compressor noise radiation. The aerodynamics of the compressor necessarily involve unsteady flow components. These unsteady flows cause unsteady forces to act upon the compressor blades so that dipole-type sound is radiated from each blade. The radiation from each blade combines to give the sound output of the rotor or stator disk, and the efficiency of propagation of this sound down the inlet and/or outlet duct is governed by the acoustic modes of the duct. The final sound which reaches the observer is the result of radiation from the end of this duct. This complexity is both an advantage and a disadvantage. It is an advantage because the many different parameters which can affect the noise radiation by the compressor can all be altered substantially independently to achieve a minimum noise radiation by the compressor. It is a disadvantage because each of these many parameters requires consideration. This introduces considerable complexity into any theoretical development or into any analysis of data from various types of engines. However, the present theory is capable of taking into account most of the effects which occur in the compressor, and it can be shown that many of the other effects are, to a large extent, negligible (ref. 14). Therefore, the theory developed is thought to have some general utility in compressor design.

**SYMBOLS**

- \( \text{B} \) number of rotor blades
- \( \text{c} \) speed of sound in undisturbed atmosphere
- \( \text{cm} \) magnitude of mth harmonic of sound radiated
- \( \text{D} \) compressor diameter
- 288
$D_k$  magnitude of $k$th torque harmonic

$F_i$  components of external force ($i = 1, 2, 3$)

$J_{\mu}$  Bessel function of first kind and of order $\mu$, where $\mu = mB - kV$

$k$  summation parameter (order of stator wake harmonic)

$M$  rotational Mach number

$m$  order of sound harmonic

$P$  sound pressure

$Q$  mass source strength

$R$  typical radius

$r$  distance from observer to source

$r_1$  distance from observer to hub

$T_{ij}$  acoustic stress tensor ($i,j = 1, 2, 3$)

$T_k$  magnitude of $k$th thrust harmonic

$t$  time

$u$  phase velocity of sound radiated

$V$  number of stator vanes

$V_T$  compressor tip velocity (mechanical tip speed)

$W$  wake velocity

$x,y$  Cartesian coordinates of observer position

$x_i,x_j$  Cartesian coordinates of source ($i,j = 1, 2, 3$)

$\theta$  angle of radiation
MECHANISMS OF NOISE GENERATION

First of all, the basic mechanisms by which the noise is generated will be reviewed. It is important to realize that sound is a wave phenomenon. Sound travels through the air at a well-defined wave speed, and because of this, sound heard at any one instant must have been generated at some earlier instant. As is illustrated in figure 2, the exact earlier time at which the sound was generated is dependent on the distance from the observer to the sound source. Thus for an extended source, sound heard at the same time \( t \) will in general have been generated at different retarded times \( \tau \) where

\[ \tau = t - \frac{r}{c} \]

Here, \( r \) represents the distance from the observer to the point under consideration and \( c \) is the speed of sound in the undisturbed atmosphere. It is therefore vitally important to account for the retarded time effects over the source if any accurate solution for the acoustic source characteristics is to be obtained.

In general, the lines of equal phase in this source region are in motion. Thus, the phase velocity in this source region becomes of considerable importance. An example is shown in figure 3. Consider that the sound radiated by some disturbance is moving with a phase velocity \( u \). From figure 3 it can be seen that \( u = \frac{c}{\cos \theta} \). Note that the phase velocity which couples into the wave is always greater than the speed of sound. As is shown in figures 3(b) and 3(c), increases in phase velocity can be accommodated by increases in the angle of radiation \( \theta \), but when the phase velocity is reduced below the speed of sound, no coupled acoustic wave is possible. For the infinitely long system illustrated here, the sound radiation from the source with a subsonic phase velocity would be identically zero. All contributions will cancel. But because the compressor has finite dimensions, cancellation is incomplete and subsonic phase velocities in the compressor can still give rise to some radiated sound. Conversely, when the phase velocity is supersonic the acoustic radiation is large. It should be noted that the phase velocity of the source is only rarely equal to its physical velocity. Figure 4 gives an example. Suppose that the two rows represent the rotor and stator of the compressor. Suppose also that a
pressure pulse is generated when any two members of the rows are in line, as shown by the arrows; this is closely analogous to the real situation. If the rotor row moves, pulses will occur as shown when the second pair comes into line. It can be seen that the effective phase velocity corresponding to the successive pulses is much higher, triple in this case, than the actual velocity of the rotor; thus both the rotor and the stator can act as "phased arrays" with a high effective velocity. Therefore, even though the rotor itself is moving at subsonic speeds, the phase velocities of the pressure fluctuations can be supersonic, and this causes efficient and undesirable radiation from the compressor. It might also be noted that when the rotor itself is moving at supersonic speeds, it can couple directly into the sound field without any requirement for these interaction effects; consequently, supersonically moving rotors are particularly efficient generators of sound. Subsonic rotors on the other hand generate significant sound levels only by the interaction process described above and are therefore more amenable to treatment at early stages of design.

Figure 5 shows, diagrammatically, the aerodynamic field existing in a compressor. Wakes stream from an initial stator and are intercepted by the rotor. As shown by the velocity diagram, the basic effect of the velocity defect in the stator wake is to cause a fluctuating downwash at the rotor disk, which will be reflected in fluctuating forces on the rotor blades. Exactly equivalent effects occur at the second stator, which experiences a fluctuating force on its vanes due to the rotor wake. Because the number of rotor and stator blades varies, the forces on the rotor or stator blades will vary in phase from blade to blade. As discussed above, these cascade phase effects are of extreme importance in determining the efficiency of the final radiation.

Several important effects related to the frequency characteristics of the compressor radiation may also be deduced from a study of figure 5. Consider first of all the rearmost stator row. Each blade is stationary and undergoes a force fluctuation due to the velocity profile of the rotor wakes. The time variation of these fluctuations is governed by the rotor speed. Thus the frequency of the noise radiated by the stator is directly related to the rotor frequency. If the rotor has \( B \) blades and rotates at angular velocity \( \Omega \), then the fundamental frequency of the stator radiation will be \( B\Omega \).

The wake velocity field behind the rotor can be analyzed into spatial Fourier components. Because of the rotation, each spatial harmonic of the rotor wake is transformed into a single temporal harmonic of the stator radiation. Now immediately behind the rotor the wake velocity profile is very sharp and thus contains all spatial frequencies, but as the wake expands downstream, it becomes smoother and less intense so that at large rotor-stator separations only the first spatial harmonic of the wake may be significant. Thus it may be predicted that the stator radiation for small rotor-stator separations will contain substantial noise at all multiples of the blade passage frequency \( B\Omega \) and that the effect of increasing rotor-stator separation will be to reduce preferentially the higher harmonics of the sound radiated by the stator.
The effects at the rotor, however, are rather different. The rotor field passes through what is essentially a stationary velocity field due to the wakes from the first stator. (See fig. 5.) The frequency of the fluctuating forces on, and thus of the acoustic radiation by, the rotor is governed by the rotor speed. Thus the angular velocity of the rotor governs temporal frequencies at both the rotor and the stator. Clearly, the stator wake can still be analyzed into spatial Fourier components, and their relative magnitude will depend on stator-rotor separation in the same way as discussed previously; that is, at large separations only the first spatial harmonic of the stator wake will be significant. Furthermore, successive spatial harmonics of the stator wake will give rise to successive loading harmonics on the rotor. However, in contrast to the stator case, each loading harmonic on the rotor gives rise to more than one sound harmonic in the radiation field.

This is due to the motion of the rotor blades. As is well known, relative motion between source and observer gives rise to Doppler frequency shifts, the observed frequency rising as the source approaches the observer and reducing as the source recedes. Thus the rotation of a particular frequency source moving with the rotor causes a periodic variation of the frequency observed at a fixed point. The effects are very similar to those of frequency-modulated radio signals. It is found that the frequency modulation causes any single frequency input to be observed as a series of frequencies, each displaced from the input frequency by some multiple of the modulation frequency. Thus fluctuating forces at one harmonic will cause radiation in all harmonics. For this reason increase of stator-rotor separation may be expected to reduce the noise basically in all the sound harmonics radiated by the rotor. This may be contrasted with the predominant reduction in the higher harmonics predicted above for the equivalent stator radiation effect.

It can be shown (ref. 14) that each loading harmonic $X$ gives rise to a different mode of radiation from the rotor, in a manner very similar to that described in the work of Tyler and Sofrin (ref. 1). Each mode will have different acoustic radiation characteristics, so that the observed radiation level from the rotor is dependent on both the acoustic radiation efficiency of each mode and the relative magnitude of its contribution from the various loading harmonics $X$. The overall radiation pattern is thus given by the sum of all the modes.

THEORETICAL DEVELOPMENT

It is not appropriate herein to give in detail the theoretical development of the equations which describe the sound radiated by the compressor. However, a brief description is justified. Any source of a sound can be described by the basic equation which was first derived by Lighthill (ref. 15) and is given as
\[
\frac{\partial^2 \rho}{\partial t^2} - c^2 \frac{\partial^2 \rho}{\partial x_1^2} = \frac{\partial Q}{\partial t} - \frac{\partial F_i}{\partial x_1} + \frac{\partial^2 T_{ij}}{\partial x_1 \partial x_j}
\]  

(1)

The left-hand side of equation (1) is the wave equation and the right-hand side can be regarded as being a collection of the various possible acoustic sources which can occur. The first term \( \frac{\partial Q}{\partial t} \) gives the effect of mass introduction. This corresponds to the siren action of the compressor which, as mentioned previously, is small. The second term \( \frac{\partial F_i}{\partial x_1} \) gives the effect of fluctuating forces. As was noted above, substantial fluctuating forces can act in the compressor because of the action of the wakes and this is the key term in the development of the present theory. The last term in equation (1) \( \frac{\partial^2 T_{ij}}{\partial x_1 \partial x_j} \) incorporates several effects, the most important of which, in general, is the direct sound radiation by turbulence. The symbol \( T_{ij} \) may be regarded as an acoustic stress tensor. Thus equation (1) can be used to describe the sound radiation by the compressor provided that the forces acting upon the compressor can be specified.

The requirement then is merely to manipulate this equation into a form which gives explicit analytical results for the sound radiation at any point. This was done in detail in reference 14. Only the final result for a typical case which is the sound radiation due to the steady and fluctuating forces acting on the rotor will be quoted herein. The expression is given as

\[
c_m = \frac{imB^2 \Omega}{2\pi c r_1} \sum_{k=-\infty}^{+\infty} (-i)^{mB-k\nu} \left( \frac{xT_k}{r_1} - \frac{mB-k\nu}{mBM} D_k \right) J_\mu \left( \frac{mBM\nu}{r_1} \right)
\]

(2)

Somewhat surprisingly an almost identical expression was found to apply to the sound radiation from the stator. Rather than describe these equations in detail, some typical results of computations using the equation will be given herein. Note particularly that all the results will apply to radiation from either rotor or stator and that both inlet and exhaust discrete-frequency radiations by a fan are included.

RESULTS AND DISCUSSION

There are two features of the sound output which are important: the directionality and the acoustic power output, both of which are predicted by the present theories. Acoustic power is the principal gross feature of the sound and is affected by the gross features of engine design, such as mass flow and revolution rate. However, directionality can also play a significant role, especially since it is more readily modified by detail design and because the sound radiated radially outward has the most significant effect on community noise. This effect can be appreciated by reference to figure 6. It can be seen that the sound radiated forward (or aft) must travel very much farther than...
that radiated downward before reaching an observer on the ground. In the case shown, there is a difference in the path length of a factor of 10. Sound is attenuated proportionally to the inverse square power of distance traveled. Thus, sound radiated along the forward path would be, in this case, 20 dB lower in intensity on reaching the ground than sound radiated downward. It is clear that community noise problems would be minimized if the sound could be chosen to radiate in an axial direction rather than radially outward. This fact is often overlooked in discussions of compressor noise. The two features, overall power and directionality, will be discussed separately subsequently.

The two key parameters are the modal order \( \mu = |n - \lambda| = |mB - kV| \), and the frequency parameter \( nM = mBM \). Essentially, the frequency parameter includes the effects of rotational speed and blade number, whereas the modal order is governed by the difference in blade numbers between rotor and stator. Note that \( nM = kR \), where \( k \) is the wave number and \( R \) a typical radius, and in this form may be more familiar to acousticians. Because of its more direct relevance to compressor parameters, \( nM \) is used herein.

For the purposes of this discussion, consider a 16-blade rotor (\( B = 16 \)) operating at \( M = 0.5 \). The solid curve in figure 7 shows the result for the effect on the first-harmonic noise (\( m = 1 \)) of the first-mode radiation (\( k = 1 \)) due to the fluctuating force terms. As has been pointed out previously the effect of large stator-rotor separation will be to emphasize the contribution of the \( k = 1 \) term, so that this is of principal interest. The curves are symmetrical about \( V = 16 \), the \( \mu = 0 \) case. Figure 7 also shows how for small numbers of stator vanes, efficiency is very low. Beyond \( V = 8 \), the acoustic radiation becomes more efficient, reaching a maximum at \( V = 10 \). Then there is a lowering of efficiency as stator vane numbers are increased up to 16. Past this point efficiency again increases up to a maximum of about 2.6 dB additional at \( V = 22 \). It may also be noted that \( V = 10 \) and \( V = 22 \) correspond to the most unfavorable side-line directivity pattern as well as the highest power, as will be shown below. For \( V = 10 \) the peak would be in front of the compressor and for \( V = 22 \), behind. For further increases in blade number, efficiency then drops off rapidly. Equivalent results will occur on all compressors. The efficient and inefficient radiation regions apparent in figure 7 correspond to supersonic and subsonic phase velocities for the interaction tones, as was discussed above.

Clearly the optimum way to utilize the results shown in figure 7 is to choose the stator vane number so that radiation is in the inefficient region of the curves. Unfortunately, this is difficult on practical compressors. Apart from the effects of multimode input and higher harmonics, the high idling Mach number of modern compressors is very restrictive. Figure 7 also shows the effect of rotational Mach number. It can be seen that the region of efficient radiation spreads out considerably and reaches down even to
zero vane numbers for $M = 1$. Thus excessive numbers of stator vanes are necessary to achieve low acoustic radiation unless the engine has been specifically designed to run at a low Mach number. Similar conclusions were reached by Tyler and Sofrin (ref. 1).

The directivity pattern of compressor noise radiation is important because of the effects discussed previously with reference to figure 6. It is desirable to radiate noise in an axial direction to minimize community noise. Figure 8 shows some directivity curves typical of the three important cases in the acoustic power plots of figure 7. Figure 8 gives the directivity factor in dB as a function of an angle from the compressor hub. The dashed lines give lines of equal side-line noise level and were calculated assuming the inverse square law and the compressor axis parallel to the ground. The sound level relative to these dashed contours will generally be the most significant parameter from the community noise point of view.

The curves in figure 8 for sound radiation are not symmetrical about the compressor disk. This is because for small values of $X (X = kV)$ the thrust and drag terms cancel in the forward quadrants but are additive in the rear quadrant, so that more sound is heard behind the compressor disk than in front. This effect is well known in propeller noise theory (ref. 16). However, as can be seen from equation (2), the situation is reversed when $X > n$; that is, when $kV > mB$. For this case more sound is radiated forward out of the compressor inlet than rearward. The effect of the $X > n$ case is given quite accurately by simply inverting the directionality patterns shown in figure 8. The effect could have practical application, since it gives a method by which the designer can choose the more intense sound to radiate forward or rearward into any available sound-absorbing device in the duct.

The two patterns on the right-hand side of figure 8 correspond to efficient radiation, whereas the radiation pattern on the left corresponds to inefficient radiation conditions. Thus, the least efficient radiation cases from the acoustic-power point of view have the worst directivity pattern from the point of view of community noise. Fortunately, the effects of the decreased efficiency will generally overcome any directivity effects. However, there are other significant features. As can be seen in figure 7, there are two maximums in the radiation efficiency. The center curve in figure 8 corresponds to this case and shows strong radial radiation. Thus it is desirable from the standpoint of both directivity and power to design compressors away from these points. Conversely, figure 7 shows how it is desirable to go to low-order modes in the center of the curves to reduce sound power. Figure 8 shows that this also has favorable directivity effects with minimal radial radiation.
COMPARISON WITH EXPERIMENT

Because of the large number of unknowns in the problem, particularly in specifications of the aerodynamic source characteristics, detailed correlation with experiment must be deferred. However, it is of interest to make some comparison herein. Figures 7 and 8 suggest the rather surprising conclusion that if the compressor had to operate in the efficient radiation region, then it would be desirable to go to equal numbers of rotor and stator blades (\( \mu = 0 \)) for minimum noise radiation. Figure 9, taken from a report by Crigler and Copeland (ref. 7) provides a verification of this prediction. Figure 9 shows that the acoustic power radiated for 53 guide vanes (\( \mu = 0 \)) is about 8 dB less than for 31 guide vanes and 7 dB less than for 62 guide vanes. Note in figure 9 how the peak levels for the \( \mu = 0 \) case are higher but carry very little acoustic power, so that power levels are low. The major reduction in sound radiation occurring for the \( \mu = 0 \) case here, particularly in the radial direction, is obviously of practical significance.

A further comparison with experiment may be accomplished by the introduction of a very simple model, discussed in more detail in reference 14. If it is assumed that the wake is of pure sinusoidal form with a momentum defect equal to the blade profile drag, then numerical values may be assigned to the fluctuating force coefficients. The fluctuating forces may be calculated by the methods of Kemp and Sears (refs. 17 and 18) as discussed in more detail in reference 14. If the asymptotic expressions for the various results are also taken, then a simple result for overall acoustic power is obtained. Using typical values for the parameters and assuming spherical spreading give the result for the side-line noise level at 200 feet as

\[
p = 50 \log_{10} V_T + 20 \log_{10} D - 75
\]

(3)

where \( V_T \) is the compressor tip velocity in \( \text{ft/sec} \) and \( D \) is the compressor diameter in inches. This curve is shown in figure 10, along with data points from several sources. In view of the large number of assumptions made it is very encouraging that the theoretical curve even falls on the data points.

The \( V_T^5 \) law implied by equation (3) has also been found in several experiments, but it is of interest that in the basic formulation (ref. 14) a \( V_R^5 \) law was derived where \( V_R \) is the relative speed of the blade as opposed to \( V_T \), the mechanical tip speed. This dependence of noise of \( V_R \) has also been suggested by several investigators. The dependence of the sound on diameter squared is also eminently reasonable. However, it should be noted that the result of equation (3) is the end result of many approximations and cannot be applied indiscriminately. The uncertainties associated with any simple approach are indicated by the wide scatter of the experimental data in figure 10. It can be seen that variations of \( \pm 10 \) dB are probable.
CONCLUDING REMARKS

A theoretical description of the noise radiated by the fluctuating forces on the rotor and stator of a fan or compressor has been derived. The theory enables the designer to account for most of the design features of the compressor. Parameters which can be included in the theory include

Rotational speed
Blade and/or vane numbers
Separation between rotor and stator
Broad features of blade and vane geometry
Stage aerodynamic and performance parameters
Multistage radiation
Compressor hub velocity

Effects which are not included in the theory include

Duct treatment
Passage of sound through blade rows
Effects of flow velocity on sound propagation
Noise radiation due to thickness or acoustic stress

The theory also suggests that the following features will not have substantial effects on the sound:

Hard-wall duct phenomena
Detailed compressor geometry

The key problem is in predicting the aerodynamic characteristics of the shed wakes. Once these are established, the theory allows, at least in principle, very comprehensive predictions of the acoustic radiation to be made. Even off-design conditions including such effects as rotating stall can be described by general theory. Preliminary agreement with experiment has been shown. Thus, it appears that the theory can be used with reasonable confidence in design trade-off studies, and it is hoped that this will lead to the design of quieter engines in the future.
REFERENCES


Figure 1.- The cause and effect chain of compressor noise radiation.

Signals received at the same instant will have been generated a time \((r_1 - r_2)/c\) apart.

Figure 2.- Effects of retarded time.
Figure 3.- Effects of source velocity.

\[ u = \frac{c}{\cos \theta} \]

Figure 4.- Rotor-stator phase effects.
BASIC WAKE GEOMETRY

Figure 5- Basic wake geometry.

Intensity $\propto \frac{1}{(\text{Distance})^2}$

Figure 6- Effect of directionality on observed noise levels.
Figure 7: Effect of rotational Mach number on acoustic power.
Figure 8.- Compressor noise directivity patterns.
Figure 9.- Measured one-third octave (at blade passage frequency) radiation patterns for various rotor – guide-vane configurations. Rotor tip Mach number $M_{tr} = 0.346$. From Crigler and Copeland (ref. 7).
Figure 10.- Comparison of "theory" and experiment. Maximum fan discharge noise in octave band containing fundamental blade passage frequency.